TfP: Teaching for PROWESS

Calculus Cognitive Interview Protocol - Students

Interviewee Name:_____Interview Date:_____

Interviewer Name:_____ Time Length (min):_____

[Before starting the interview, be sure the student was provided with the consent form and ask for their signature. For a virtual interview, this form should be emailed in advance, signed, scanned, and emailed back to the interviewer.]

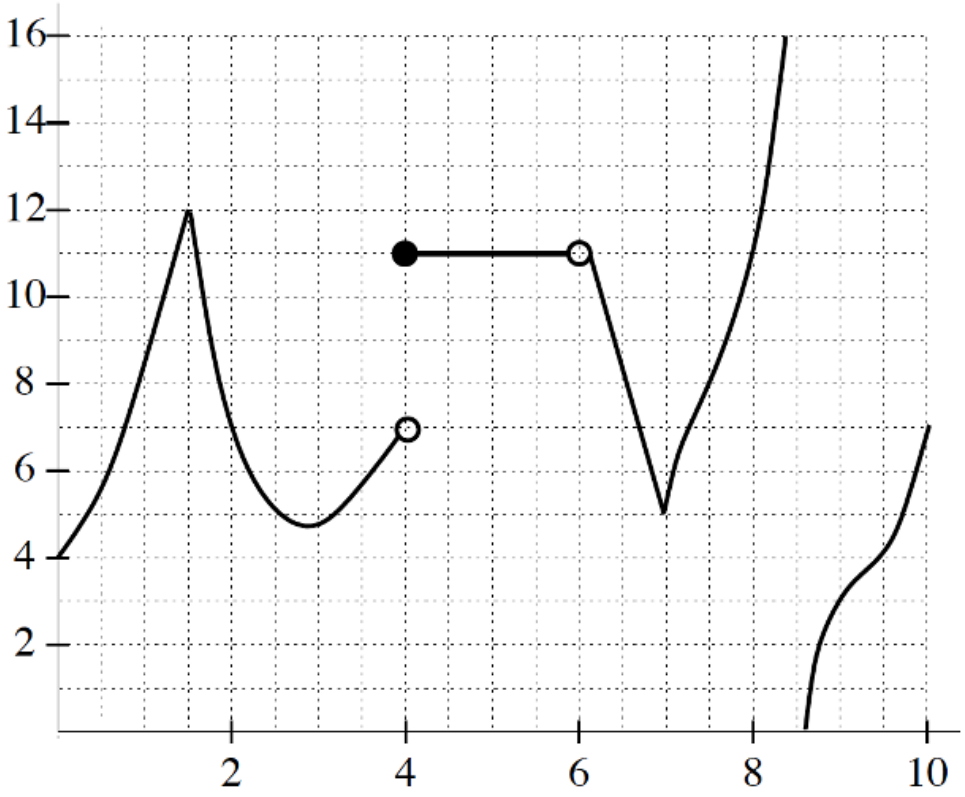
Thank you for agreeing to participate in this interview. I am <interviewer name> and I serve as part of the research team for the NSF-funded Teaching for PROWESS project. Data are being collected as part of a research project for the Teaching for PROWESS project to better understand the development of mathematical ideas presented in Calculus. As part of our research we are interested in gaining knowledge about how you approach various mathematical tasks. We will present mathematical tasks and will ask you questions to get at your thinking on a deeper level. As we progress through the interview, we ask that you verbalize all your thoughts so that we can gather information about your thinking on these tasks.

*This interview should take no more than **one hour.** The information you provide will be kept confidential, and no names will be used in our reports. Your participation in this interview is voluntary, so you can choose to answer all of the questions, some of the questions, or none of the questions, without any negative impacts for you. If at any time you wish to stop the interview, please let me know. We want you to be as comfortable as possible.*

As noted in the consent form that you just signed, I plan to record this interview so that we can be sure to capture your responses verbatim, rather than relying on handwritten notes. Is that ok? [Turn recorder on] Before we begin, do you have any questions about the project, your participation, or anything else?

Task #1:

Consider the graph of the function $y = f(x)$ given below, then answer the following questions:



1. At what values of x is the function f not continuous? Explain how you know.
2. At what values of x is the function f not differentiable? Explain how you know.

Task #2:

When the Discovery space shuttle is launched, its speed increases continually until its booster engines separate from the shuttle. During the time it is continually speeding up, the shuttle is never moving at a constant speed. What, then, would it mean to say that at precisely 2.15823 seconds after launch the shuttle is traveling at precisely 183.8964 miles per hour?

Task #3:

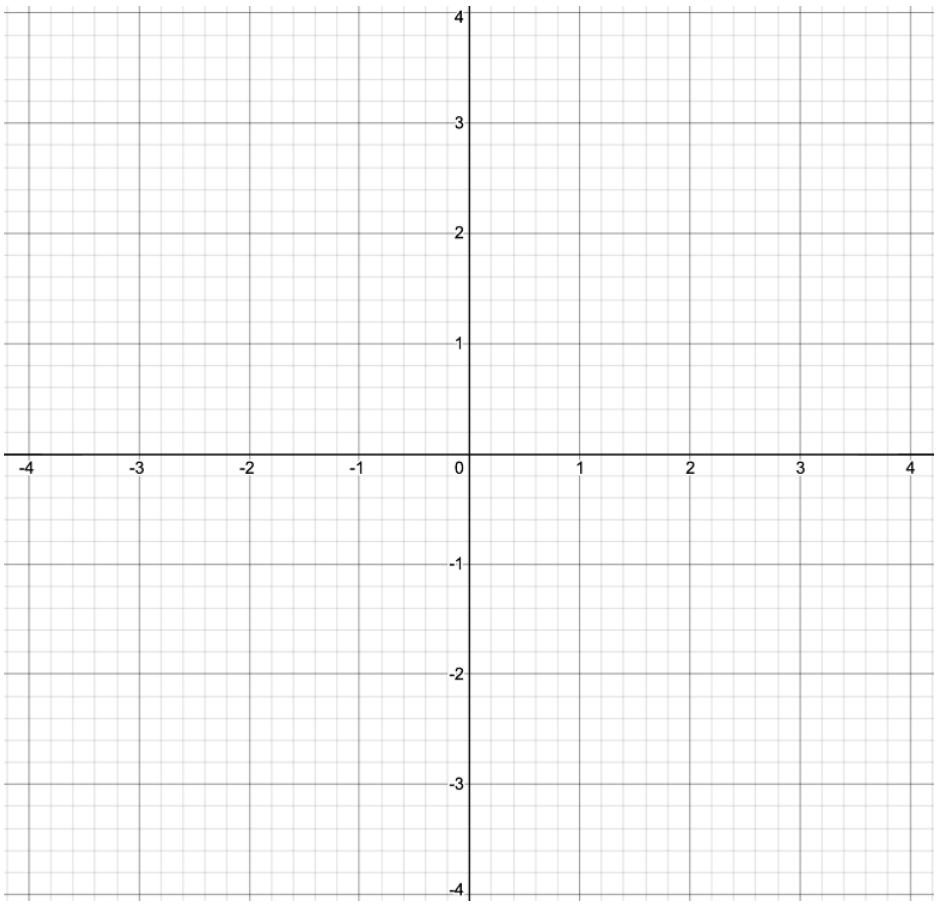
Sketch a graph of a continuous function *f* with the following properties and explain your process as you sketch the graph.

$f'(x) > 0$ **over the intervals** $x < -3$ **and** $x > 2$

$f'(x) < 0$ **over the interval** $(-3, 2)$

$f''(x) < 0$ **over the interval** $(-\infty, -1)$

$f''(x) > 0$ **over the interval** $(-1, \infty)$



Task #4:

Given that time (t) is measured in years, the function C(t) represents the per capita consumption of soda, in gallons per person, which is currently about 39 gallons per person and is decreasing by about 0.4 gallons per person each year. The function P(t) represents the population of the United States which is currently 327 million people and increasing by 1.5 million people per year. Estimate how the total consumption of soda is changing.

Task #5:

Big Tex wants to fabricate a cylindrical BBQ offset smoker, with the main barrel having a volume of 8000 cubic inches (ignore the offset smoker box on the side). The steel for the main barrel part of the cylinder costs $0.21 per square inch and the steel for the 2 ends of the barrel costs $0.14 per square inch. What is the cost of the least expensive main barrel part of the cylindrical BBQ offset smoker that Big Tex can build (ignore that the smoker will need other parts to work fully)? Use calculus to find your solution, and be sure to also include a fully labeled diagram that connects to your solution!

Task #6:

In a time of *t* seconds, a particle moves a distance of *s* meters from its starting point, where $s = f(t) = t^2 + 1$.

1. Find the average velocity between $t = 2$ and $t = 2 + h$ if $h = 0.1$, $h = 0.01$, and $h = 0.001$. (That is, compute the average velocity over 3 different time intervals).

2. Now, give your best estimate of the instantaneous velocity of the particle at $t = 2$.

3. Using $h = 0.1$,illustrate on a graph where one can find the following quantities:
   a. $t = 2$
   b. $t = 2 + h$
   c. $f(2)$
   d. $f(2 + h)$
   e. How can you use the graph to make sense of average rate of change between $(2, f(2))$ and $((2 + h), f(2 + h))$.