

*Please read the entire Introduction before completing the Rubric*

The Teaching for PROWESS (TfP) Vision & Transformation Catalyst Tool\* is a diagnostic tool designed to be used in a self-study to evaluate the implementation of the recommendations of the AMATYC Standards (referring to [Crossroads in Mathematics](#), [Beyond Crossroads](#), and [IMPACT](#)) in mathematics departments. The work is based on the extensive work of Partnership for Undergraduate Life Science Education (PULSE)\*\* which was focused on Biology in 4-year institutions. They have been modified based on the features expected in a 2-year college math department that has fully implemented all of the AMATYC recommendations. They are meant as tools to highlight the areas where departments stand out and areas where departments have made less progress.

The complete Teaching for PROWESS Vision & Transformation Catalyst Tool contains 8 rubrics:

**1) Student Learning and the Learning Environment, 2) Instruction, 3) Curriculum and Program Development, 4) Assessment of Student Learning, 5) Diversity, Equity, and Inclusion, 6) Professionalism, 7) Climate for Transformation and 8) Snapshot.**

**Terminology:** The rubrics can be used to evaluate individual departments, or a division composed of mathematics faculty (either full-time or part-time) which will be referred to as 'departments' in this document. The use of the term 'faculty' throughout the rubrics is meant as a generic term for the range of possible titles for all those who are instructors in any course that is part of the department being evaluated.

**Procedure:** Once a department chooses an area, or areas, they would like to examine, the faculty should then individually determine scores for the rubrics. Each criterion begins with a **CONTEXT** section that should be read *prior to* reading the criterion's descriptors. Once a score for a criterion is determined it is important to document the justification in the appropriate section of the table. After the individual results are completed, the department should determine and report a consensus score for each criterion. For more information and suggestions on completing this process, refer to the Rubric FAQs on the [teachingforprowess.wordpress.com](http://teachingforprowess.wordpress.com) website.

\* This material is based upon work supported by the National Science Foundation under Grants No. 2012962, 2013232, 2013493, 2013550. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

\*\* An initiative launched by the National Science Foundation (NSF), the Howard Hughes Medical Institute (HHMI), and the National Institute for General Medical Sciences (NIGMS/NIH).

### **Rubric III - Curriculum and Program Development (15 criteria)**

This rubric assesses the extent to which mathematics departments develop, implement, evaluate, assess, and revise courses, course sequences, and programs to help students attain a higher level of quantitative literacy, better quantitative reasoning skills, and achieve their academic and career goals. Mathematics departments, in collaboration with appropriate stakeholders, should regularly engage in course and program review and evaluation (refer to Assessment of Student Learning rubric) to assure that the mathematics curricula 1) prepare students to be numerate citizens and productive employees who have an appreciation for mathematics and lifelong learning and 2) meet the mathematical needs of client disciplines. These course and program reviews and the subsequent revisions should reflect the department's own analysis of student achievement and the informed practice of the mathematics community. Categories include: A) Standards for Content and B) Standards for Intellectual Development. We use the term practices to describe what students are engaged in to be competent doers and users of mathematical content and ways of thinking.

## A. STANDARDS FOR CONTENT

Rather than focusing solely on content knowledge, this document takes the position that to truly know mathematics and statistics one must know it conceptually, contextually, and procedurally and that problem solving is the heart of doing mathematics. The successful problem solver can view the world from a mathematical perspective (Schoenfeld, 1992).

Students develop the ability to solve meaningful problems through in-depth study of mathematics and statistics topics that build on their prior knowledge and experiences. When presented in the context of relevant applications, abstract topics grow naturally out of the need to describe or represent the patterns that emerge. In general, the meaning, use, and communication of mathematical and statistical ideas must be emphasized. Attention to rote memorization and manipulation must decrease.

The content standards that follow are not meant to outline a set of courses. Rather, they are strands to be included in a post-secondary mathematics pathway in whatever structural form they may take. The specific themes were selected so that learners can develop the knowledge and skills needed to be discerning citizens, making data-based decisions, and evaluating mathematical and statistical arguments. Students should also be equipped to pursue more advanced study in mathematics and other disciplines.

### **CRITERION A1:** Integration of numeracy into the curriculum

Students will accurately perform arithmetic operations, and will process, interpret, and communicate numerical information.

“Numeracy is the ability to process, interpret, and communicate numerical, quantitative, spatial, statistical, even mathematical, information, in ways that are appropriate for a variety of contexts, and that will enable a typical member of the culture or subculture to participate effectively in activities that they value.” (Evans, 2000) Students should be able to identify and perform appropriate arithmetic operations, estimate reliably, judge the reasonableness of numerical results, understand orders of magnitude, think proportionally, and make sense of data (especially large data sets) to recognize patterns. This mathematical reasoning may be enhanced through the use of technology.

A		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
1	<b>Integration of numeracy into the curriculum</b>	Numeracy practices are not included in any courses	Minimal integration of numeracy practices in the curriculum	Numeracy practices are an explicit focus of at least one required course	Numeracy practices are an explicit focus across <i>several</i> required courses	Numeracy practices are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification A1 (Required):

## A. STANDARDS FOR CONTENT

**CRITERION A2:** Integration of symbolism and algebra into the curriculum

Students will be able to interpret algebraic symbols, translate problems into appropriate symbolic representations, and use those representations to effectively answer questions and make predictions.

Students will move beyond concrete numerical operations and use algebraic thinking and symbols to solve problems. Students will represent mathematical situations using a combination of appropriate symbolic, graphical, and numerical methods to form conjectures about the problems. Applications of algebraic thinking include derivation of formulas, translation of realistic problems into mathematical statements, conversion between different representations, and the solution of equations by appropriate methods.

A		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
2	<b>Integration of symbolism and algebra into the curriculum</b>	Symbolizing practices are not included in any courses	Minimal integration symbolizing practices in the curriculum	Symbolizing practices are an explicit focus of at least one required course	Symbolizing practices are an explicit focus across <i>several</i> required courses	Symbolizing practices are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification A2 (Required):

## A. STANDARDS FOR CONTENT

### CRITERION A3: Integration of geometry and measurement into the curriculum

Students will develop a spatial and measurement sense, learn to visualize and use geometric models, recognize measurable attributes, and use and convert units of measure.

Geometry is the study of visual patterns. Every physical object has a shape, so every physical object is geometric. Furthermore, mathematical objects can be represented geometrically. For example, real numbers are represented on a number line, forces are represented with vectors, and statistical distributions are represented with the graphs of curves. The use of dynamic geometry software provides for efficient integration of geometric concepts throughout the curriculum, allowing students to more effectively visualize geometric representations.

Students will demonstrate their abilities to visualize, compare, and transform objects using geometric representations. Students will develop a spatial sense including the ability to draw (either by hand or with the use of technology) one-dimensional, two-dimensional, and three-dimensional shapes from different perspectives, and extend a concept, such as vectors, to higher dimensions. Their knowledge of geometry will enable them to determine dimensions, area, perimeter, and volume of common plane and solid figures. Suggested topics might include comparison of geometric objects (including congruence and similarity), graphing, prediction from graphs, measurement, and vectors.

A		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
3	<b>Integration of geometry and measurement into the curriculum</b>	Spatial and measurement are not included in any courses	Minimal spatial and measurement in the curriculum	Spatial and measurement are an explicit focus of at least one required course	Spatial and measurement are an explicit focus across <i>several</i> required courses	Spatial and measurement are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification A3 (Required):

## A. STANDARDS FOR CONTENT

### CRITERION A4: Integration of functions into the curriculum

Students will demonstrate understanding of the concept of function by several means - numerically, graphically, symbolically, and verbally - and incorporate it as a central theme into their use of mathematics.

During the last decade, several key curricular issues have stimulated dialogue and educational research. For example, since 1989, when it was suggested that “If it does nothing else, undergraduate mathematics should help students develop function sense...,” (National Research Council, 1989), considerable research has been conducted on what it means for students to have an understanding of function. Studies report that a well-developed understanding of function correlates closely with success in calculus, as well as facilitating the transition to advanced mathematical thinking (Tall, 1992; Carlson (1998)). In addition, faculty continue to search for methods to develop a student’s understanding of the concept of variable. Students who are able to view variables as representing quantities whose values change dynamically along a continuum have been shown to have ready access to fundamental ideas, such as rate of change and limits, and exhibit higher levels of achievement in mathematics. (Ursini, S., & Trigueros, M. (1997), Jacobs, S. (2002))

In regards to functions: students will know when a relation is a function; students will use function notation and perform operations on functions; and students will interpret functional relationships between two or more variables, formulate such relationships when presented in tabular, graphical, symbolic, or verbal representations and convert between representations. Suggested topics include generalization about families of functions, transformations of functions, use of functions to model realistic problems, and the behavior of functions.

A		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
4	<b>Integration of functions into the curriculum</b>	Functional representation practices are not included in any courses	Minimal integration of functional representation practices in the curriculum	Functional representation practices are an explicit focus of at least one required course	Functional representation practices are an explicit focus across <i>several</i> required courses	Functional representation practices are an explicit focus in <i>most</i> required courses and students’ use of these practices is assessed

Justification A4 (Required):

## A. STANDARDS FOR CONTENT

**CRITERION A5:** Integration of discrete mathematics into the curriculum

Students will be able to recognize and use discrete mathematical algorithms and develop combinatorial abilities in order to solve problems of finite character and enumerate sets without direct counting.

This standard provides guidance for incorporating topics from discrete mathematics courses (which may require precalculus or calculus as prerequisites) into introductory college mathematics courses. Applications in the social and behavioral sciences, business, computing, and other areas frequently do not exhibit the continuous nature commonly treated by techniques studied in introductory college mathematics pathways. Rather, these applications involve discrete objects and focus on logic and enumeration (Dossey, 1991; Hart, 1991). The standard echoes the recommendations made in the NCTM Standards (NCTM, 2008) and in Reshaping College Mathematics (Siegel, 1989); namely, the conceptual framework of discrete mathematics should be integrated throughout the introductory mathematics pathways, as appropriate, in order to improve students' problem-solving skills and prepare them for the study of higher levels of mathematics as well as for their careers. Suggested topics in discrete mathematics may include set theory, logic, graph theory, game theory, algorithms, sequences, series, permutations, combinations, recursion, linear programming, finite graphs, voting systems, and matrices.

A		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
5	<b>Integration of discrete mathematics into the curriculum</b>	Combinatorial practices are not included in any courses	Minimal integration of combinatorial practices in the curriculum	Combinatorial practices are an explicit focus of at least one required course	Combinatorial practices are an explicit focus across <i>several</i> required courses	Combinatorial practices are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification A5 (Required):

## A. STANDARDS FOR CONTENT

**CRITERION A6:** Integration of statistics and probability into the curriculum

Students will use data to inform decisions and understand the world around them.

The basic concepts of statistics, data science, and probability should be integrated throughout the curriculum using relevant contexts and appropriate technology. Students should recognize and describe variability, take variability into account when making decisions, and make and communicate data-based arguments. Suggested topics include appropriate methods for collecting data, creating and interpreting data visualizations, sampling variability, drawing conclusions from sample data, modeling, applications of probability, and the ethical use of data.

A		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
6	<b>Integration of statistics and probability into the curriculum</b>	Statistical and probability practices are not included in any courses	Minimal integration statistical and probability practices in the curriculum	Statistical and probability practices are an explicit focus of at least one required course	Statistical and probability practices are an explicit focus across <i>several</i> required courses	Statistical and probability practices are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification A6 (Required):

## A. STANDARDS FOR CONTENT

**CRITERION A7:** Integration of deductive proof into the curriculum

Students will appreciate the deductive nature of mathematics as an identifying characteristic of the discipline; recognize the roles of definitions, axioms, and theorems; and identify and construct valid deductive arguments.

The use of deductive proof in mathematics sets it apart as a unique area of human endeavor. Where appropriate to enhance student understanding of mathematical concepts, mathematical proofs, including indirect proofs and mathematical induction, will be introduced. Students will engage in exploratory activities that will lead them to form statements of conjecture, test them by seeking counterexamples, and identify and, in some instances, construct arguments verifying or disproving the statements. A variety of proof techniques, including the use of manipulatives, diagrams, and pictures to create proofs without words or symbols, should also be encouraged.

A		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
7	<b>Integration of deductive proof into the curriculum</b>	Deductive proving practices are not included in any courses	Minimal integration of deductive proving practices in the curriculum	Deductive proving practices are an explicit focus of at least one required course	Deductive proving practices are an explicit focus across <i>several</i> required courses	Deductive proving practices are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification A7 (Required):



## B. STANDARDS FOR INTELLECTUAL DEVELOPMENT

At the conclusion of the first two years of their college studies, all students should have progressed in their development of certain intellectual abilities, habits of mind, and of other competencies and knowledge. Introductory college courses across disciplines should be designed to broaden an existing educational foundation and allow students to appreciate mathematics, statistics, and data science as powerful reasoning, modeling, and general problem solving tools.

**CRITERION B1:** Inclusion of problem solving throughout the curriculum

Students will engage in relevant, authentic problem solving and mathematical and statistical thinking.

Students will use problem-solving strategies that require persistence, analysis of assumptions, intellectual risk taking, and application of appropriate procedures. These strategies should include posing questions; collecting and organizing information; constructing visual representations; solving similar, simpler problems; analyzing situations through trial and error, graphing, and modeling; and drawing conclusions by translating, illustrating, and verifying results. The students should be able to communicate and interpret their results.

Emphasizing problem solving will make mathematics more meaningful to students. The problems used should be relevant to the needs and interests of the students in the class. Such problems provide a context as well as a purpose for learning new skills, concepts, and theories.

B		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
1	<b>Inclusion of problem solving throughout the curriculum</b>	Problem solving practices are not included in any courses	Minimal integration of problem solving practices in the curriculum	Problem solving practices are an explicit focus of at least one required course	Problem solving practices are an explicit focus across <i>several</i> required courses	Problem solving practices are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification B1 (Required):

## B. STANDARDS FOR INTELLECTUAL DEVELOPMENT

**CRITERION B2:** Inclusion of modeling throughout the curriculum

Students will learn mathematics and statistics through modeling real-world situations.

Students will participate in the mathematical and statistical modeling of situations from the world around them and use the models to make predictions and informed decisions. Swetz (1991) describes the mathematical modeling process as "(1) identifying the problem, including the conditions and constraints under which it exists; (2) interpreting the problem mathematically; (3) employing the theories and tools of mathematics to obtain a solution to the problem; (4) testing and interpreting the solution in the context of the problem; and (5) refining the solution techniques to obtain a 'better' answer to the problem under consideration, if necessary" (pp. 358-359). The statistical modeling process is similar but also involves connecting data, chance, and context (Pfannkuch, et.al, 2018).

Whether students develop their own models or evaluate models that are given to them, they should look beyond how well a proposed model fits a set of data and attempt to provide contextual, mathematical, statistical, or data-based reasons for why the model is valid.

B		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
2	<b>Inclusion of modeling throughout the curriculum</b>	Modeling practices are not included in any courses	Minimal integration of modeling practices in the curriculum	Modeling practices are an explicit focus of at least one required course	Modeling practices are an explicit focus across <i>several</i> required courses	Modeling practices are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification B2 (Required):

## B. STANDARDS FOR INTELLECTUAL DEVELOPMENT

**CRITERION B3:** Inclusion of reasoning throughout the curriculum

Students will expand their mathematical and statistical reasoning skills as they develop convincing mathematical, statistical, and data-based arguments.

Students will regularly apply inductive and deductive reasoning techniques to build convincing mathematical, statistical, and/or data-based arguments. They will develop conjectures on the basis of previous knowledge, data, and intuition and test these conjectures by using logic and deductive and inductive proof, by framing examples and counterexamples, and by probabilistic and statistical reasoning. They will then draw appropriate conclusions and communicate their argument convincingly. In addition, students will judge the validity of mathematical, statistical, and/or data-based arguments using the same reasoning skills.

B		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
3	<b>Inclusion of reasoning throughout the curriculum</b>	Mathematical reasoning practices are not included in any courses	Minimal integration of mathematical reasoning practices in the curriculum	Mathematical reasoning practices are an explicit focus of at least one required course	Mathematical reasoning practices are an explicit focus across <i>several</i> required courses	Mathematical reasoning practices are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification B3 (Required):

## B. STANDARDS FOR INTELLECTUAL DEVELOPMENT

**CRITERION B4:** Connecting with other disciplines is expected throughout the curriculum

Students will develop the view that mathematics, statistics, and data science are growing disciplines, are interrelated with human culture, and understand their connections to other disciplines.

If students are to gain a sense that mathematics, statistics, and data science are growing disciplines, course content must include current and relatable topics such as algorithms needed for computer-based solution processes, the use of probability in understanding chance and randomization, modern approaches to statistical inference and data visualization, and the applications of non-Euclidean geometries. These topics lend themselves to discussions of who developed the ideas, when they were developed, and what kind of human endeavors motivated their development, which reinforces recognition of math in all parts of life and cultures. Students should develop an appreciation of how mathematics and statistics provide a language for the sciences; play a role in art, music, and literature; are applied by social scientists and practitioners in health care fields; are used in business and manufacturing; and have impacted history.

B		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
4	<b>Connecting with other disciplines is expected throughout the curriculum</b>	Appreciation of mathematics is not included in any courses	Minimal integration of appreciation of mathematics in the curriculum	Appreciation of mathematics is an explicit focus of at least one required course	Appreciation of mathematics is an explicit focus across <i>several</i> required courses	Appreciation of mathematics is an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification B4 (Required):

## B. STANDARDS FOR INTELLECTUAL DEVELOPMENT

**CRITERION B5:** Communicating is expected throughout the curriculum

Students will develop the ability to read, write, listen to, and speak the languages of mathematics, statistics, and data science.

Students will develop the skills necessary to communicate ideas and procedures, and results using appropriate mathematical and statistical vocabulary and notation. Students will develop the ability to communicate the results of analyses through appropriate models and visualizations. Furthermore, mathematics, statistics, and data science faculty will adopt instructional strategies that develop both oral and written communication skills within a context of authentic applications relevant to a diverse student population. As students learn to speak and write about mathematics, statistics, and data science, they develop acumen and become better prepared to use this knowledge and these skills beyond the classroom.

B		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
5	<b>Communicating is expected throughout the curriculum</b>	Mathematical communication practices are not included in any courses	Minimal integration of mathematical communication practices in the curriculum	Mathematical communication practices are an explicit focus of at least one required course	Mathematical communication practices are an explicit focus across <i>several</i> required courses	Mathematical communication practices are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification B5 (Required):

## B. STANDARDS FOR INTELLECTUAL DEVELOPMENT

**CRITERION B6:** Using technology is expected throughout the curriculum

Students will use appropriate technology to enhance their thinking and conceptual understanding and to solve problems.

Students will develop an ability to use technology to enhance their study of mathematics, statistics, and data science. Current technology can be used to aid in the understanding, exploration, and visualization of concepts and the analysis of data. Students can use technology to test conjectures, explore ideas, and verify that theorems are true in specific instances. They should also embrace technology as a tool to aid in the solution of authentic problems and to validate those solutions. Students should be able to judge the reasonableness and accuracy of the results generated by technology.

		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
6	<b>Using technology is expected throughout the curriculum</b>	Appropriate use of technology is not included in any courses	Minimal integration appropriate use of technology in the curriculum	Appropriate use of technology is an explicit focus of at least one required course	Appropriate use of technology is an explicit focus across <i>several</i> required courses	Appropriate use of technology is an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification B6 (Required):

## B. STANDARDS FOR INTELLECTUAL DEVELOPMENT

**CRITERION B7:** Developing mathematical prowess is expected throughout the curriculum

Students will engage in rich experiences in the study of mathematics, statistics, data science, and related fields that encourage independent, nontrivial exploration and will develop and reinforce tenacity and confidence in their abilities and inspire them to further their studies in these fields.

Students will develop self-confidence and persistence while engaging with mathematics, statistics, and data science problem-solving. These problems will not always have unique solutions but will provide experiences that develop the ability to conduct independent explorations. At the same time, they will learn to transfer problem-solving strategies to a variety of contexts (Druckman & Bjork, 1994) and appreciate mathematics, statistics, and data science as disciplines. They will visualize themselves using mathematics and statistics effectively in their professional work and everyday lives. They will develop an awareness of careers in mathematics and related disciplines.

B		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
7	<b>Developing mathematical prowess is expected throughout the curriculum</b>	Independent, nontrivial exploration is not included in any courses	Minimal integration of Independent, nontrivial exploration in the curriculum	Independent, nontrivial exploration is an explicit focus of at least one required course supporting students' developing confidence and tenacity with mathematical practices	Independent, nontrivial exploration is an explicit focus across <i>several</i> required courses	Numeracy practices are an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification B7 (Required):

## B. STANDARDS FOR INTELLECTUAL DEVELOPMENT

**CRITERION B8:** Linking multiple representations is expected throughout the curriculum

Students will select, use, and translate among mathematical and statistical representations—numerical, graphical, symbolic, and verbal—to organize information and solve problems using a variety of techniques.

Students will explore complex problems, using multiple approaches, and explain their solutions in both oral and written form. Students will be motivated to go beyond the mastery of basic operations, statistical algorithms, or algebraic manipulations to a real understanding of how to use mathematics and statistics, the meaning of the answers, and how to interpret them.

B		(0) Baseline	(1) Beginning	(2) Developing	(3) Accomplished	(4) Exemplar
8	<b>Linking multiple representations is expected throughout the curriculum</b>	Linking multiple representations is not included in any courses	Minimal integration of linking multiple representations in the curriculum	Linking multiple representations is an explicit focus of at least one required course; supports students' developing confidence and tenacity with mathematical practices	Linking multiple representations is an explicit focus across <i>several</i> required courses	Linking multiple representations is an explicit focus in <i>most</i> required courses and students' use of these practices is assessed

Justification B8 (Required):